DESY 95-157
hep-ph/yyyyy
August 1995

Rare Radiative B Decays in the Standard Model

Ahmed Ali
Deutsches Elektronen-Synchrotron DESY, D-22603 Hamburg

Abstract

A status report on the theory and phenomenology of rare radiative B decays in the standard model is presented with emphasis on the measured decays $B \to X_s\gamma$ and $B \to K^*\gamma$. Standard model is in agreement with experiments though this comparison is not completely quantitative due to imprecise data and lack of the complete next-to-leading order contributions to the decay rates. Despite this, it is possible to extract non-perturbative parameters from the shape of the photon energy spectrum in $B \to X_s\gamma$, such as the b-quark mass and the kinetic energy of the b quark in B hadron. The measured decay rate $\mathcal{B}(B \to X_s + \gamma) = (2.32 \pm 0.67) \times 10^{-4}$ can also be used to extract the CKM ratio $|V_{ts}|/|V_{cb}|$, yielding $|V_{ts}|/|V_{cb}| = 1.10 \pm 0.43$. Issues bearing on the determination of the parameters of the CKM matrix from the CKM-suppressed decays $B \to X_d + \gamma$ and $B \to (\rho, \omega) + \gamma$ are also discussed. It is argued that valuable and independent constraints on the CKM matrix can be obtained from the measurements of these decays, in particular those involving neutral B-mesons.

to appear in the Proceedings of the XXXth Rencontres de Moriond
"Electroweak Interactions and Unified Theories"
Les Arcs, France, March 1995

1 Introduction

The last two years have seen the first observations of the electromagnetic penguins in B decays by the CLEO collaboration. These include the measurements of the exclusive decay rate, $\mathcal{B}(B \to K^\star + \gamma) = (4.5 \pm 1.0 \pm 0.9) \times 10^{-5}$ [1], and the inclusive rate $\mathcal{B}(B \to X_s + \gamma) = (2.32 \pm 0.67) \times 10^{-4}$ [2], yielding $R(K^*/X_s) \equiv \Gamma(B \to K^\star + \gamma)/\Gamma(B \to X_s + \gamma) = 0.19 \pm 0.09$. In addition, the charged and neutral B-meson decay rates are found equal within experimental measurements. The inclusive decay rate is in agreement with the predictions of the standard model [3, 4, 5], with the rate estimated as $\mathcal{B}(B \to X_s + \gamma) = (2.55 \pm 1.28) \times 10^{-4}$ assuming $|V_{ts}|/|V_{cb}| = 1.0$ [6]. Conversely, one can vary the Cabibbo-Kobayashi-Maskawa (CKM) matrix element ratio and determine it from $\mathcal{B}(B \to X_s + \gamma)$, which yields $|V_{ts}|/|V_{cb}| = 1.10 \pm 0.43$. The ratio $R(K^*/X_s)$ is well explained by the QCD-sum-rule based estimates of the recent vintage [7, 8] and by wave-function models combined with vector meson dominance (local parton-hadron duality) [3]. This and the near equality of the charged and neutral decay rates imply that the observed radiative B decays are dominated by the (common) electromagnetic penguin (short distance) amplitudes and the contributions from B-meson-specific diagrams (weak annihilation, $W^\pm$-exchange) are small.

The photon energy spectrum in $B \to X_s + \gamma$ yields information on the structure function of the photon in the electromagnetic penguins [9, 10, 11, 12]. In specific models [13, 14], this information can be transcribed in terms of non-perturbative parameters, such as the b-quark mass and the kinetic energy of the b quark in B hadron. These quantities can also be estimated in the framework of the heavy quark effective theory [15] combined with QCD sum rules [16, 17]. The results of a recent analysis of the CLEO data on $B \to X_s + \gamma$ [6] are consistent with the values expected from such theoretical considerations. However, this agreement is presently not completely quantitative due to imprecise data.

There is considerable interest in measuring the CKM-suppressed radiative B decays, such as $B \to X_d + \gamma$ [18] and $B \to (\rho, \omega) + \gamma$ [7]. A determination of the CKM parameters from eventual measurements of these decays requires careful treatment of the competing short-distance (SD) and long-distance (LD) effects. This problem can be formulated in terms of model-independent correlation functions involving matrix elements of a few dimension-6 operators in an effective theory. Techniques, such as the QCD sum rules, can then be invoked to estimate them. In [19, 20], the leading LD-effects in the exclusive decays $B \to (\rho, \omega) + \gamma$ are calculated in terms of the weak annihilation amplitudes. The largest such effects may show themselves in the charged $B^\pm$-decays, $B^\pm \to \rho^\pm + \gamma$, contributing up to $O(15\%)$ of the corresponding SD-amplitudes; their influence in the neutral B-decays is estimated to be much smaller. Hence, there are good theoretical reasons to plead that the decays $B^0 \to (\rho^0, \omega) + \gamma$ and $B \to X_d + \gamma$ are well suited to determine the CKM parameters. We take up these issues in this status report.

2 Estimates of $\mathcal{B}(B \to X_s + \gamma)$ in the SM

The framework that is used generally to discuss the decays $B \to X_s + \gamma$ is that of an effective theory with five quarks, obtained by integrating out the heavier degrees of freedom, which in the standard model are the top quark and the W-boson. A complete set of dimension-6 operators relevant for the processes $b \to s + \gamma$ and $b \to s + \gamma + g$ is contained in the effective

Hamiltonian

$$H_{eff}(b \to s\gamma) = -\frac{4G_F}{\sqrt{2}}\,\lambda_t \sum_{j=1}^{8} C_j(\mu)\,O_j(\mu) \quad , \tag{1}$$

where G_F is the Fermi constant coupling constant, $C_j(\mu)$ are the Wilson coefficients evaluated at the scale μ, and $\lambda_t = V_{tb}V_{ts}^*$ with V_{ij} being the CKM matrix elements. The overall multiplicative factor λ_t follows from the CKM unitarity and neglecting λ_u. The operators O_j read

$$\begin{aligned}
O_1 &= (\bar{c}_{L\beta}\gamma^\mu b_{L\alpha})\,(\bar{s}_{L\alpha}\gamma_\mu c_{L\beta})\ , \\
O_2 &= (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})\,(\bar{s}_{L\beta}\gamma_\mu c_{L\beta})\ , \\
O_3 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})\left[(\bar{u}_{L\beta}\gamma_\mu u_{L\beta}) + ... + \left(\bar{b}_{L\beta}\gamma_\mu b_{L\beta}\right)\right] , \\
O_4 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta})\left[(\bar{u}_{L\beta}\gamma_\mu u_{L\alpha}) + ... + \left(\bar{b}_{L\beta}\gamma_\mu b_{L\alpha}\right)\right] , \\
O_5 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})\left[(\bar{u}_{R\beta}\gamma_\mu u_{R\beta}) + ... + \left(\bar{b}_{R\beta}\gamma_\mu b_{R\beta}\right)\right] , \\
O_6 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta})\left[(\bar{u}_{R\beta}\gamma_\mu u_{R\alpha}) + ... + \left(\bar{b}_{R\beta}\gamma_\mu b_{R\alpha}\right)\right] , \\
O_7 &= (e/16\pi^2)\,\bar{s}_\alpha\,\sigma^{\mu\nu}\,(m_b(\mu)R + m_s(\mu)L)\,b_\alpha\ F_{\mu\nu}\ , \\
O_8 &= (g_s/16\pi^2)\,\bar{s}_\alpha\,\sigma^{\mu\nu}\,(m_b(\mu)R + m_s(\mu)L)\,(\lambda^A_{\alpha\beta}/2)\,b_\beta\ G^A_{\mu\nu} \quad ,
\end{aligned} \tag{2}$$

where e and g_s are the electromagnetic and the strong coupling constants, respectively. In the magnetic moment type operators O_7 and O_8, $F_{\mu\nu}$ and $G^A_{\mu\nu}$ denote the electromagnetic and the gluonic field strength tensors, respectively. The subscripts on the quark fields $L \equiv (1-\gamma_5)/2$ and $R \equiv (1+\gamma_5)/2$ denote the left and right-handed projection operators, respectively. QCD corrections to the decay rate for $b \to s\gamma$ bring in large logarithms of the form $\alpha_s^n(m_W)\,\log^m(m_b/M)$, where $M = m_t$ or m_W and $m \leq n$ (with $n = 0, 1, 2, ...$). Using the renormalization group equations the Wilson coefficient can be calculated at the scale $\mu \approx m_b$ which is the relevant scale for B decays. To leading logarithmic precision, it is sufficient to know the leading order anomalous dimension matrix and the matching $C_i(\mu = m_W)$ to lowest order (i.e., without QCD corrections) [21]. The 8×8 anomalous dimension matrix is given in [4], from where references to earlier calculations can also be obtained, the Wilson coefficients are explicitly listed in [5] and the numerical values of these coefficients being used here can be seen in [6].

It has become customary to calculate the branching ratio for the radiative decay $B \to X_s + \gamma$ in terms of the semileptonic decay branching ratio $\mathcal{B}(B \to X\ell\nu_\ell)$

$$\mathcal{B}(B \to X_s\gamma) = [\frac{\Gamma(B \to X_s + \gamma)}{\Gamma_{sl}}]\,R(m_b,\mu)\,\mathcal{B}(B \to X\ell\nu_\ell) \qquad , \tag{3}$$

where, in the approximation of including the leading-order QCD correction, Γ_{sl} is given by the expression

$$\Gamma_{sl} = \frac{G_F^2\,|V_{cb}|^2}{192\pi^3}\,m_b^5\,g(m_c/m_b)\,(1 - 2/3\frac{\alpha_s}{\pi}f(m_c/m_b)) \quad . \tag{4}$$

The phase space function $g(z)$ and the function $f(z)$ due to one-loop QCD corrections can be seen in [13]. The radiative decay rate $\Gamma(B \to X_s + \gamma)$ are worked out in [3, 6], taking into account $O(\alpha_s)$ virtual and bremsstrahlung corrections. In calculating the matrix elements in these papers, the on-shell subtraction prescription for the quark masses has been used. Due

Parameter	Range
$\overline{m_t}$ (GeV)	170 ± 11
μ (GeV)	$5.0^{+5.0}_{-2.5}$
Λ_5 (GeV)	$0.195^{+0.065}_{-0.05}$
$\mathcal{B}(B \to X \ell \nu_\ell)$	$(10.4 \pm 0.4)\%$
m_c/m_b	0.29 ± 0.02
m_W (GeV)	80.33
$\alpha^{-1}_{\mathrm{QED}}$	130.0

Table 1: Values of the parameters used in estimating the branching ratio $\mathcal{B}(B \to X_s + \gamma)$ in the standard model.

to the explicit factors of the running quark masses in the operators O_7 and O_8, the m_b^5-factor contained in the decay rate $\Gamma(B \to X_s + \gamma)$ is replaced by the following product

$$m_b^5 \longrightarrow m_b(pole)^3 \ m_b(\mu)^2 \quad , \tag{5}$$

where $m_b(pole)$ and $m_b(\mu)$ denote the pole mass and the $\overline{\mathrm{MS}}$-running mass of the b quark, respectively. Since, in the leading order in α_s, the semileptonic decay width Γ_{sl} depends on the product $m_b(pole)^5$, the ratio of the two decay widths brings in the correction factor $R(m_b, \mu)$:

$$R(m_b, \mu) = [m_b(\mu)/m_b(pole)]^2 \quad , \tag{6}$$

as also remarked in [5]. At the one-loop level, these masses are related:

$$\frac{m_b(\mu)}{m_b(pole)} = [\frac{\alpha_s(\mu)}{\alpha_s(m_b)}]^{4/\beta_0}[1 - \frac{4}{3}\frac{\alpha_s(\mu)}{\pi}], \tag{7}$$

where $\beta_0 = 23/3$. The parameters used in estimating the inclusive rates for $\mathcal{B}(B \to X_s + \gamma)$ are summarized in table 1.

We now discuss $\mathcal{B}(B \to X_s + \gamma)$ in the standard model and theoretical uncertainties on this quantity [6].

- Scale dependence of the Wilson coefficients.

The largest theoretical uncertainty stems from the scale dependence of the Wilson coefficients. As derived explicitly in [6], the decay rate for $B \to X_s + \gamma$ depends on seven of the eight Wilson coefficients in $H_{eff}(b \to s)$, once one takes into account the bremsstrahlung corrections and is not factored in terms of a single (effective) coefficient, namely C_7^{eff}, that one encounters for the two-body decays $b \to s + \gamma$ [4, 5]. Numerical values of the two dominant effective coefficients, C_7^{eff} and C_8^{eff}, as one varies μ, the QCD scale Λ_5, and the (running) top quark mass in the $\overline{\mathrm{MS}}$-scheme $\bar{m}_t(m_t)$ in the range given in table 1, are:

$$\begin{aligned} C_7^{eff} &\equiv C_7 - \frac{C_5}{3} - C_6 = -0.306 \pm 0.050, \\ C_8^{eff} &\equiv C_8 + C_5 = -0.146 \pm 0.020. \end{aligned} \tag{8}$$

This is the dominant theoretical error on $\mathcal{B}(B \to X_s + \gamma)$, contributing about $\pm 35\%$.

- Scale-dependence of $m_b(\mu)$ in the operators O_7 and O_8.

This brings into fore the extra (scale-dependent) multiplicative factor $R(m_b, \mu)$ for the branching ratio $\mathcal{B}(B \to X_s + \gamma)$, as discussed above. Intrinsic uncertainties in the concept of the pole mass due to infrared renormalons suggest that one should express all physical results in terms of the running masses [22]. This requires recalculating the decay rate $\mathcal{B}(B \to X_s + \gamma)$ with the running masses, incorporating resummations of the kind recently undertaken for the semileptonic B decay rates [23].

- Extrinsic errors in $\mathcal{B}(B \to X_s + \gamma)$

The next largest error arises from the parameters which are extrinsic to the decay $B \to X_s + \gamma$ and have crept in due to normalizing the branching ratio $\mathcal{B}(B \to X_s + \gamma)$ in terms of $\mathcal{B}(B \to X \ell \nu_\ell)$. The first of these extrinsic errors is related to the uncertainty in the ratio m_c/m_b. Using for the b quark pole mass $m_b(pole) = 4.8 \pm 0.15$ GeV [16] and $m_b - m_c = 3.40$ GeV [24], one gets $m_c/m_b = 0.29 \pm 0.02$. Taking into account the experimental error of $\pm 4.1\%$ on $\mathcal{B}(B \to X \ell \nu_\ell)$ [25], one estimates an extrinsic error of $\pm 12\%$ on $\mathcal{B}(B \to X_s + \gamma)$.

Assuming $|V_{ts}|/|V_{cb} = 1$ [26], the branching ratio $\mathcal{B}(B \to X_s \gamma)$ calculated as a function of the top quark mass is shown in Fig. 1 [6]. For all three solid curves the quark mass ratio is

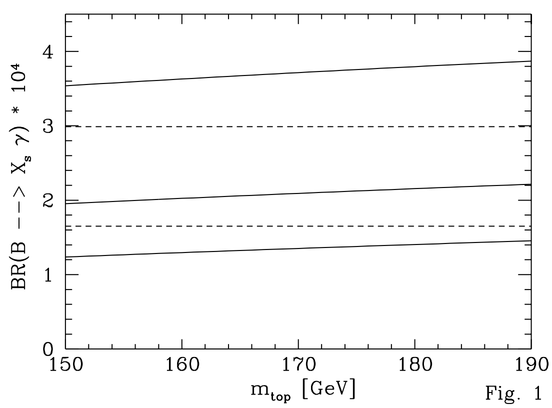

Figure 1: $\mathcal{B}(B \to X_s \gamma)$ as a function of the $\overline{\text{MS}}$ top quark mass. The three solid lines correspond to the variation of the parameters μ and Λ_5 as described in the text. The experimental ($\pm 1\sigma$)-bounds from CLEO [2] are shown by the dashed lines (from [6]).

fixed at $m_c/m_b = 0.29$. The top solid curve is drawn for $\mu = 2.5$ GeV and $\Lambda_5 = 0.260$ GeV. The bottom solid curve is for $\mu = 10$ GeV and $\Lambda_5 = 0.145$ GeV, and the middle solid curve corresponds to the central values of the input parameters in table 1. Using $\overline{m}_t = (170 \pm 11)$ GeV, and adding the extrinsic error, one obtains:

$$\mathcal{B}(B \to X_s + \gamma) = (2.55 \pm 1.28) \times 10^{-4}\,, \tag{9}$$

to be compared with the CLEO measurement $\mathcal{B}(B \to X_s + \gamma) = (2.32 \pm 0.67) \times 10^{-4}$. The ($\pm 1\sigma$)-upper and -lower bound from the CLEO measurement are shown in Fig. 1 by dashed lines. The agreement between SM and experiment is good, given the large uncertainties on both. In [6], the branching ratio $\mathcal{B}(B \to X_s + \gamma)$ has been calculated as a function of the CKM matrix element ratio squared $(|V_{ts}|/|V_{cb}|)^2$, varying $\bar{m}_t$, μ and Λ_5 in the range specified in table 1. Using the ($\pm 1\sigma$)-experimental bounds on $\mathcal{B}(B \to X_s + \gamma)$, one infers [6]:

$$|V_{ts}|/|V_{cb}| = 1.10 \pm 0.43\,, \tag{10}$$

which is consistent with the indirect constraints from the CKM unitarity [26] yielding $|V_{ts}|/|V_{cb}| \simeq 1.0$ but imprecise. Further improvements require reducing the perturbative scale(μ)-dependence of the decay rate, which in turn implies calculations of the next-to-leading order terms, and more accurate measurements.

3 Photon Energy Spectrum in $B \to X_s + \gamma$

The two-body partonic process $b \to s\gamma$ yields a photon energy spectrum $1/\Gamma d\Gamma(b \to s\gamma) = \delta(1-x)$, where the scaled photon energy x is defined as $E_\gamma = (m_b^2 - m_s^2)/(2\, m_b)\, x$; x then varies in the interval $[0,1]$. Perturbative QCD corrections, such as $b \to s\gamma + g$, give a characteristic bremsstrahlung spectrum peaking near the end-points, $E_\gamma \to E_\gamma^{max}$ and $E_\gamma \to 0$, arising from the soft-gluon and soft-photon configurations, respectively. As long as the s-quark mass is non-zero, there is no collinear singularity in the spectrum. Near the end-points, one has to improve the spectrum obtained in fixed order perturbation theory. This is usually done by isolating and exponentiating the leading behaviour in $\alpha_{em}\alpha_s(\mu)^m \log^n(1-x)$ and $\alpha_{em}\alpha_s(\mu)^m \log^n x$, with $m \leq n$, where μ is a typical momentum in the decay $B \to X_s + \gamma$. The running of α_s is a non-leading effect, but as it is characteristic of QCD it modifies the Sudakov-improved end-point photon energy spectrum [11] compared to its analogue in QED [27]. Away from the end-points, the photon energy spectrum has to be calculated completely in a given order in α_s in perturbation theory [3, 6].

The complete photon energy spectrum in $B \to X_s + \gamma$ is at present not calculable in QCD from first principles. The situation is very much analogous to that of other hadronic structure functions. It has been observed in a number of papers [9, 10, 11], that the x-moments of the inclusive photon energy spectrum in $B \to X_s + \gamma$ and those of the lepton energy spectrum in the decay $B \to X_u \ell \nu_\ell$ are related. Defining the moments as:

$$\begin{aligned} \mathcal{M}_n(B \to X_s + \gamma) &\equiv \frac{1}{\Gamma}\int_0^{M_B/m_b} dx x^{n-1} \frac{d\Gamma}{dx} \\ \mathcal{M}_n(B \to X_u \ell \nu_\ell) &\equiv -\int_0^{M_B/m_b} dx x^n \frac{d}{dx}\left(\frac{1}{\Gamma_\ell}\frac{d\Gamma_\ell}{dx}\right) \\ &= \frac{n}{\Gamma_\ell}\int_0^{M_B/m_b} dx x^{n-1} \frac{d\Gamma_\ell}{dx}\ , \end{aligned} \tag{11}$$

the ratios of the moments are free of non-perturbative complications. The moments $\mathcal{M}_n$ have been worked out in the leading non-trivial order in perturbation theory and the results can be expressed as:

$$\mathcal{M}_n \sim 1 + \frac{\alpha_s}{2\pi} C_F (A \log^2 n + B \log n + \text{const.}) \tag{12}$$

where $C_F = 4/3$, the leading coefficient is universal with $A = -1$ [27], and the non-leading coefficients are process dependent; $B = 7/2$ [3] and $B = 31/6$ [28], for $B \to X_s + \gamma$ and $B \to X_u \ell \nu_\ell$, respectively. Measurements of the moments could eventually be used to relate the CKM matrix element V_{ts} and V_{ub}. That this method will give competitive values for V_{ub}, however, depends on whether or not the coefficient functions in $\Gamma(B \to X_s + \gamma)$ discussed in the previous section are known to the desired level of theoretical accuracy.

We shall leave such theoretically improved comparisons for future Rencontres de Moriond and confine ourselves to the discussion of the present state-of-the-art comparison of the measured photon energy spectrum in $B \to X_s + \gamma$ with the perturbative QCD-improved treatment of the same. The analysis that we discuss here [3, 6] treats the non-perturbative effects in

terms of a B-meson wave function. In this model [13], which admittedly is simplistic but not necessarily wrong, the b quark in B hadron is assumed to have a Gaussian distributed Fermi motion determined by a non-perturbative parameter, p_F,

$$\phi(p) = \frac{4}{\sqrt{\pi} {p_F}^3} \exp(\frac{-p^2}{{p_F}^2}) \quad , \quad p = |\vec{p}| \tag{13}$$

with the wave function normalization $\int_0^\infty dp\, p^2\, \phi(p) = 1$. The photon energy spectrum from the decay of the B-meson at rest is then given by

$$\frac{d\Gamma}{dE_\gamma} = \int_0^{p_{max}} dp\, p^2\, \phi(p) \frac{d\Gamma_b}{dE_\gamma}(W, p, E_\gamma) \quad , \tag{14}$$

where p_{max} is the maximally allowed value of p and $\frac{d\Gamma_b}{dE_\gamma}$ is the photon energy spectrum from the decay of the b-quark in flight, having a momentum-dependent mass $W(p)$.

An analysis of the CLEO photon energy spectrum has been undertaken in [6] to determine the non-perturbative parameters of this model, namely $m_b(pole)$ and p_F. The experimental errors are still large and the fits result in relatively small χ^2 values; the minimum, $\chi^2_{min} = 0.038$, is obtained for $p_F = 450$ MeV and $m_b(pole) = 4.77$ GeV, in good agreement with theoretical estimates of the same, namely $m_b(pole) = 4.8 \pm 0.15$ GeV [16] and $p_F^2 = \mu_\pi^2/2 = 0.25 \pm 0.05$ GeV^2 obtained from the QCD sum rules [17]. In Fig. 2 we have plotted the photon energy spectrum normalized to unit area in the interval between 1.95 GeV and 2.95 GeV for the parameters which correspond to the minimum χ^2 (solid curve) and for another set of parameters that lies near the χ^2-boundary defined by $\chi^2 = \chi^2_{min} + 1$. (dashed curve). Data from CLEO [2] are also shown. Further details of this analysis can be seen in [6].

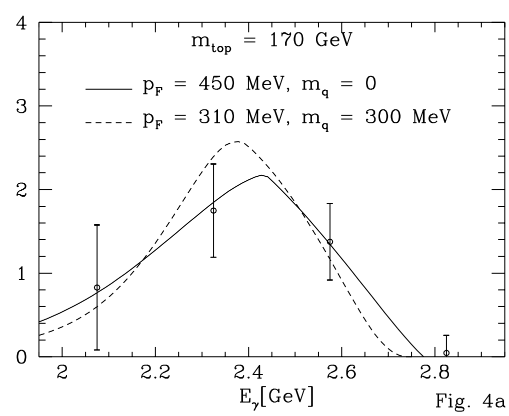

Figure 2: Comparison of the normalized photon energy distribution using the CLEO data [2] corrected for detector effects and theoretical distributions from [6] , both normalized to unit area in the photon energy interval between 1.95 GeV and 2.95 GeV. The solid curve corresponds to the values with the minimum χ^2, (m_q, p_F)=(0,450 MeV), and the dashed curve to the values (m_q, p_F)=(300 MeV, 310 MeV).

4 Inclusive radiative decays $B \to X_d + \gamma$

The theoretical interest in the standard model in the (CKM-suppressed) inclusive radiative decays $B \to X_d + \gamma$ lies in the first place in the possibility of determining the CKM-Wolfenstein

parameters ρ and η [30]. The relevant region in the decays $B \to X_d + \gamma$ is the end-point photon energy spectrum, which has to be measured requiring that the hadronic system X_d recoiling against the photon does not contain strange hadrons to suppress the large-E_γ photons from the decay $B \to X_s + \gamma$. Assuming that this is feasible, one can determine from the ratio of the decay rates $\mathcal{B}(B \to X_d + \gamma)/\mathcal{B}(B \to X_s + \gamma)$ the CKM-Wolfenstein parameters. This measurement was proposed in [18], where the final-state spectra were also worked out.

In close analogy with the $B \to X_s + \gamma$ case discussed earlier, the complete set of dimension-6 operators relevant for the processes $b \to d\gamma$ and $b \to d\gamma g$ can be written as:

$$H_{eff}(b \to d) = -\frac{4G_F}{\sqrt{2}}\,\xi_t \sum_{j=1}^{8} C_j(\mu)\,\hat{O}_j(\mu), \tag{15}$$

where $\xi_j = V_{jb}V_{jd}^*$ for $j = t, c, u$. The operators $\hat{O}_j,\ j = 1, 2$, have implicit in them CKM factors. In the Wolfenstein parametrization [30], one can express these factors as : $\xi_u = A\,\lambda^3\,(\rho - i\eta),\ \xi_c = -A\,\lambda^3,\ \xi_t = -\xi_u - \xi_c$. We note that all three CKM-angle-dependent quantities ξ_j are of the same order of magnitude, $O(\lambda^3)$, where $\lambda = \sin\theta_C \simeq 0.22$. This is an important difference as compared to the effective Hamiltonian $\mathcal{H}_{eff}(b \to s)$ written earlier, in which case the effective Hamiltonian factorizes into an overall CKM factor λ_t. For calculational ease, this difference can be implemented by defining the operators $\hat{O}_1$ and $\hat{O}_2$ entering in $H_{eff}(b \to d)$ as follows [18]:

$$\begin{aligned}
\hat{O}_1 &= -\frac{\xi_c}{\xi_t}(\bar{c}_{L\beta}\gamma^\mu b_{L\alpha})(\bar{d}_{L\alpha}\gamma_\mu c_{L\beta}) - \frac{\xi_u}{\xi_t}(\bar{u}_{L\beta}\gamma^\mu b_{L\alpha})(\bar{d}_{L\alpha}\gamma_\mu u_{L\beta}),\\
\hat{O}_2 &= -\frac{\xi_c}{\xi_t}(\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{d}_{L\beta}\gamma_\mu c_{L\beta}) - \frac{\xi_u}{\xi_t}(\bar{u}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{d}_{L\beta}\gamma_\mu u_{L\beta}),
\end{aligned} \tag{16}$$

and the rest of the operators ($\hat{O}_j;\ j = 3...8$) are defined like their counterparts O_j in $H_{eff}(b \to s)$, with the obvious replacement $s \to d$. With this definition, the matching conditions $C_j(m_W)$ and the solutions of the RG equations yielding $C_j(\mu)$ become identical for the two operator basis O_j and $\hat{O}_j$. It has been explicitly checked in the $O(\alpha_s)$ calculations of the decay rate and photon energy spectrum involving $b \to dg$ and $b \to dg\gamma$ transitions that the limit $m_u \to 0$ for the decay rate $\Gamma(B \to X_d + \gamma)$ exists [18]. From this it follows that, in the leading order QCD corrections, there are no logarithms of the type $\alpha_s \log(m_u^2/m_c^2)$ [29]. Some papers, estimating LD-contributions in radiative B decays, seem to contradict this by assuming light-quark contributions which have such spurious log-dependence. There is no calculational basis for this assumption. In higher orders, such terms must be absorbed in the non-perturbative functions. On the other hand, as far as the dependence of the decay rate and spectra on the external light quark masses is concerned, one encounters logarithms of the type $\alpha_{em}\alpha_s(1+(1-x)^2)/x\log(m_b^2/m_s^2)$ (for $b \to sg\gamma$) and $\alpha_{em}\alpha_s(1+(1-x)^2)/x\log(m_b^2/m_d^2)$ (for $b \to dg\gamma$) near the soft-photon ($x \to 0$) region [6], which can, however, be exponentiated [12]. The essential difference between $\Gamma(B \to X_s + \gamma)$ and $\Gamma(B \to X_d + \gamma)$ lies in the matrix elements of the first two operators O_1 and O_2 (in $H_{eff}(b \to s)$) and $\hat{O}_1$ and $\hat{O}_2$ (in $H_{eff}(b \to d)$). The derivation of the inclusive decay rate and the final-state distributions in $B \to X_d + \gamma$ otherwise goes along very similar lines as for the decays $B \to X_s + \gamma$. The branching ratio $\mathcal{B}(B \to X_d + \gamma)$ in the SM can be written as:

$$\begin{aligned}
&\mathcal{B}(B \to X_d + \gamma) = D_1|\xi_t|^2\\
&\{1 - \frac{1-\rho}{(1-\rho)^2+\eta^2}\,D_2 - \frac{\eta}{(1-\rho)^2+\eta^2}\,D_3 + \frac{D_4}{(1-\rho)^2+\eta^2}\},
\end{aligned} \tag{17}$$

where the functions D_i depend on the parameters listed in table 1. The uncertainty on this branching ratio from the parametric dependence is very similar to the one worked out for $\mathcal{B}(B \to X_s + \gamma)$. For the central values of the parameters in table 1, one gets : $D_1 = 0.21$, $D_2 = 0.17$, $D_3 = 0.03$, $D_4 = 0.10$. To get the inclusive branching ratio the CKM parameters ρ and η have to be constrained from the unitarity fits. Taking the parameters from a recent fit, one gets $5.0 \times 10^{-3} \leq |\xi_t| \leq 1.4 \times 10^{-2}$ (at 95% C.L.) [31], yielding an order of magnitude uncertainty in $\mathcal{B}(B \to X_d + \gamma)$ - hence the interest in measuring it. Taking the central values of the fit parameters $A = 0.8, \lambda = 0.2205, \eta = 0.34$ and $\rho = -0.07$ [31], one gets $\mathcal{B}(B \to X_d + \gamma) = (1.7 \pm 0.85) \times 10^{-5}$, which is approximately a factor 10 -20 smaller than the CKM-allowed branching ratio $\mathcal{B}(B \to X_s + \gamma)$, measured by CLEO [1].

5 Estimates of $\mathcal{B}(B \to V + \gamma)$ and Constraints on the CKM Parameters ρ and η

Exclusive radiative B decays $B \to V + \gamma$, with $V = K^*, \rho, \omega$, are also potentially very interesting from the point of view of determining the CKM parameters [7]. The extraction of these parameters would, however, involve a trustworthy estimate of the SD- and LD-contributions in the decay amplitudes.

The SD-contribution in the exclusive decays $(B_u, B_d) \to (K^*, \rho) + \gamma$, $B_d \to \omega + \gamma$ and the corresponding B_s decays, $B_s \to (\phi, K^*) + \gamma$, involve the magnetic moment operator O_7 and the related one obtained by the obvious change $s \to d$, $\hat{O}_7$. The transition form factors governing the radiative B decays $B \to V + \gamma$ can be generically defined as:

$$\langle V, \lambda | \frac{1}{2} \bar{\psi} \sigma_{\mu\nu} q^\nu b | B \rangle = i \epsilon_{\mu\nu\rho\sigma} e_\nu^{(\lambda)} p_B^\rho p_V^\sigma F_S^{B \to V}(0). \tag{18}$$

Here V is a vector meson with the polarization vector $e^{(\lambda)}$, $V = \rho, \omega, K^*$ or ϕ; B is a generic B-meson B_u, B_d or B_s, and ψ stands for the field of a light u, d or s quark. The vectors p_B, p_V and $q = p_B - p_V$ correspond to the 4-momenta of the initial B-meson and the outgoing vector meson and photon, respectively. In (18) the QCD renormalization of the $\bar{\psi} \sigma_{\mu\nu} q^\nu b$ operator is implied. Keeping only the SD-contribution leads to obvious relations among the exclusive decay rates, exemplified here by the decay rates for $(B_u, B_d) \to \rho + \gamma$ and $(B_u, B_d) \to K^* + \gamma$:

$$\frac{\Gamma((B_u, B_d) \to \rho + \gamma)}{\Gamma((B_u, B_d) \to K^* + \gamma)} = \frac{|\xi_t|^2}{|\lambda_t|^2} \frac{|F_S^{B \to \rho}(0)|^2}{|F_S^{B \to K^*}(0)|^2} \Phi_{u,d} = \kappa_{u,d} [\frac{|V_{td}|}{|V_{ts}|}]^2 , \tag{19}$$

where $\Phi_{u,d}$ is a phase-space factor which in all cases is close to 1 and $\kappa_i \equiv [F_S^{B_i \to \rho\gamma} / F_S^{B_i \to K^* \gamma}]^2$ is the ratio of the (SD) form factors squared. The transition form factors F_s are model dependent. However, their ratios, i.e. κ_i, should be more reliably calculable as they depend essentially only on the SU(3)-breaking effects. If the SD-amplitudes were the only contributions, the measurements of the CKM-suppressed radiative decays $(B_u, B_d) \to \rho + \gamma$, $B_d \to \omega + \gamma$ and $B_s \to K^* + \gamma$ could be used in conjunction with the decays $(B_u, B_d) \to K^* + \gamma$ to determine the CKM parameters. The present experimental upper limits on the CKM ratio $|V_{td}|/|V_{ts}|$ from radiative B decays are indeed based on this assumption, yielding [32]:

$$\frac{|V_{td}|}{|V_{ts}}| \leq 0.75 , \tag{20}$$

with a theoretical dispersion estimated in the range 0.64 - 0.75, depending on the models used for the $SU(3)$ breaking effects in the form factors [7, 33].

The possibility of significant LD- contributions in radiative B decays from the light quark intermediate states has been raised in a number of papers [34]. Their amplitudes necessarily involve other CKM matrix elements and hence the simple factorization of the decay rates in terms of the CKM factors involving $|V_{td}|$ and $|V_{ts}|$ no longer holds thereby invalidating the relationships given above. In what follows, we argue that the CKM-analysis of charged B-decays, $B^\pm \to \rho^\pm\gamma$, would require modifications due to the LD-contributions but the corresponding analysis of the neutral B-decays $B \to (\rho^0, \omega)\gamma$ remains essentially unchanged.

The LD-contributions in $B \to V + \gamma$ are induced by the matrix elements of the four-Fermion operators $\hat{O}_1$ and $\hat{O}_2$ (likewise O_1 and O_2). Estimates of these contributions require non-perturbative methods. This problem has been investigated recently in [19, 20] using a technique which treats the photon emission from the light quarks in a theoretically consistent and model-independent way. This has been combined with the light-cone QCD sum rule approach to calculate both the SD and LD — parity conserving and parity violating — amplitudes in the decays $B_{u,d} \to \rho(\omega) + \gamma$. To illustrate this, we concentrate on the $B_u^\pm$ decays, $B_u^\pm \to \rho^\pm + \gamma$ and take up the neutral B decays $B_d \to \rho(\omega) + \gamma$ at the end. The LD-amplitude of the four-Fermion operators $\hat{O}_1$, $\hat{O}_2$ is dominated by the contribution of the weak annihilation of valence quarks in the B meson. It is color-allowed for the decays of charged $B^\pm$ mesons, as shown in fig. 3, where also the tadpole diagram is shown, which, however, contributes only in the presence of gluonic corrections, and hence neglected. In the factorization approximation, one may write the dominant contribution in the operator $\hat{O}_2$ (here O_2' is the part of $\hat{O}_2$ with the CKM factor ξ_u/ξ_t)

$$\langle \rho\gamma | O_2' | B \rangle = \langle \rho | \bar{d}\Gamma_\mu u | 0 \rangle \langle \gamma | \bar{u}\Gamma^\mu b | B \rangle + \langle \rho\gamma | \bar{d}\Gamma_\mu u | 0 \rangle \langle 0 | \bar{u}\Gamma^\mu b | B \rangle \ , \tag{21}$$

and make use of the definitions of the decay constants

$$\begin{aligned} \langle 0 | \bar{u}\Gamma_\mu b | B \rangle &= i p_\mu f_B, \\ \langle \rho | \bar{d}\Gamma_\mu u | 0 \rangle &= \varepsilon_\mu^{(\rho)} m_\rho f_\rho, \end{aligned} \tag{22}$$

to reduce the problem at hand to the calculation of simpler form factors induced by vector and axial-vector currents.

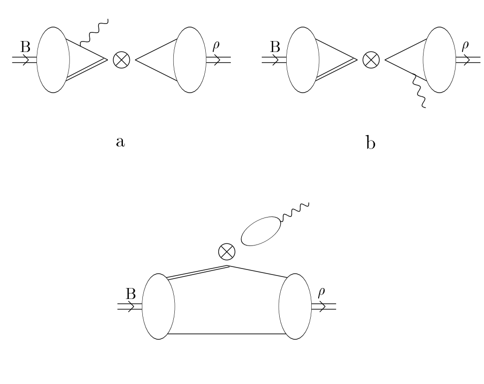

Figure 3: Weak annihilation contributions in $B_u \to \rho\gamma$ involving the operators O_1' and O_2' denoted by $\otimes$ with the photon emission from a) the loop containing the b quark, b) the loop containing the light quark, and c) the tadpole which contributes only with additional gluonic corrections.

The factorization approximation assumed in [19, 20] has been tested (to some extent) in two-body and quasi-two body non-leptonic B decays involving the transitions $b \to c\bar{c}s$ and

$b \to c\bar{u}d$. It has not been tested experimentally in radiative B decays. From a theoretical point of view, non-factorizable contributions belong to either the $O(\alpha_s)$ (and higher order) radiative corrections or to contributions of higher-twist operators to the sum rules. Their inclusion should not change the conclusions substantially.

The LD-amplitude in the decay $B_u \to \rho^\pm + \gamma$ can be written in terms of the form factors F_1^L and F_2^L,

$$\begin{aligned}\mathcal{A}_{long} &= -\frac{e\,G_F}{\sqrt{2}} V_{ub}V_{ud}^* \left(C_2 + \frac{1}{N_c}C_1\right) m_\rho \varepsilon_\mu^{(\gamma)}\varepsilon_\nu^{(\rho)} \\ &\times \Big\{ -i\Big[g_{\mu\nu}(q\cdot p) - p_\mu q_\nu\Big]\cdot 2F_1^L(q^2) + \epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \cdot 2F_2^L(q^2)\Big\}\,. \end{aligned} \tag{23}$$

Again, one has to invoke a model to calculate the form factors. Estimates from the light-cone QCD sum rules give [19]:

$$F_1^L/F_S = 0.0125 \pm 0.0010\,, \quad F_2^L/F_S = 0.0155 \pm 0.0010\,, \tag{24}$$

where the errors correspond to the variation of the Borel parameter in the QCD sum rules. Including other possible uncertainties, one expects an accuracy of the ratios in (24) of order 20%. Since the parity-conserving and parity-violating amplitudes turn out to be close to each other, $F_1^L \simeq F_2^L \equiv F_L$, the ratio of the LD- and the SD- contributions reduces to a number

$$\mathcal{A}_{long}/\mathcal{A}_{short} = R_{L/S}^{B_u\to\rho\gamma}\cdot\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}\,. \tag{25}$$

Using $C_2 = 1.10$, $C_1 = -0.235$, $C_7^{eff} = -0.306$ (at the scale $\mu = 5$ GeV) [6] gives:

$$R_{L/S}^{B_u\to\rho\gamma} \equiv \frac{4\pi^2 m_\rho (C_2 + C_1/N_c)}{m_b C_7^{eff}}\cdot\frac{F_L^{B_u\to\rho\gamma}}{F_S^{B_u\to\rho\gamma}} = -0.30 \pm 0.07\,. \tag{26}$$

To get a ball-park estimate of the ratio $\mathcal{A}_{long}/\mathcal{A}_{short}$, we take the central values of the CKM matrix elements, $V_{ud} = 0.9744 \pm 0.0010$ [26], $|V_{td}| = (1.0 \pm 0.2)\times 10^{-2}$, $|V_{cb}| = 0.039 \pm 0.004$ and $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ [31], yielding,

$$|\mathcal{A}_{long}/\mathcal{A}_{short}|^{B_u\to\rho\gamma} = |R_{L/S}^{B_u\to\rho\gamma}|\frac{|V_{ub}V_{ud}|}{|V_{td}V_{bt}|} \simeq 10\%\,. \tag{27}$$

The analogous LD-contributions to the neutral B decays $B_d \to \rho\gamma$ and $B_d \to \omega\gamma$ are expected to be much smaller, a point that has also been noted in the context of the VMD and quark model based estimates [34]. In the present approach, the corresponding form factors for the decays $B_d \to \rho^0(\omega)\gamma$ are obtained from the ones for the decay $B_u \to \rho^\pm\gamma$ discussed above by the replacement of the light quark charges $e_u \to e_d$, which gives the factor $-1/2$; in addition, and more importantly, the LD-contribution to the neutral B decays is colour-suppressed, which reflects itself through the replacement of the factor $a_1 \equiv C_2 + C_1/N_c$ in (23) by $a_2 \equiv C_1 + C_2/N_c$. This yields for the ratio

$$\frac{R_{L/S}^{B_d\to\rho\gamma}}{R_{L/S}^{B_u\to\rho\gamma}} = \frac{e_d a_2}{e_u a_1} \simeq -0.13 \pm 0.05, \tag{28}$$

where the numbers are based on using $a_2/a_1 = 0.27 \pm 0.10$ [35]. This would then yield at most $R_{L/S}^{B_d\to\rho\gamma} \simeq R_{L/S}^{B_d\to\omega\gamma} = 0.05$, which in turn gives $\mathcal{A}_{long}^{B_d\to\rho\gamma}/\mathcal{A}_{short}^{B_d\to\rho\gamma} \leq 0.02$. Even if this

underestimates the LD-contribution by a factor 2, due to the approximations made in [19, 20], it is quite safe to neglect the LD-contribution in the neutral B-meson radiative decays.

The ratio of the CKM-suppressed and CKM-allowed decay rates for charged B mesons gets modified due to the LD contributions. Following [36], we ignore the LD-contributions in $\Gamma(B \to K^*\gamma)$. The ratio of the decay rates in question can therefore be written as:

$$\frac{\Gamma(B_u \to \rho\gamma)}{\Gamma(B_u \to K^*\gamma)} = \kappa_u \lambda^2[(1-\rho)^2 + \eta^2] \times \left\{1 + 2 \cdot R_{L/S} V_{ud} \frac{\rho(1-\rho) - \eta^2}{(1-\rho)^2 + \eta^2} + (R_{L/S})^2 V_{ud}^2 \frac{\rho^2 + \eta^2}{(1-\rho)^2 + \eta^2}\right\}, \tag{29}$$

Using the central value from the estimates of the ratio of the form factors squared $\kappa_u = 0.59 \pm 0.08$ [7], and the presently allowed range of the Wolfenstein parameters ρ and η, it is shown in [19] that the effect of the LD-contributions is modest but not negligible, introducing an uncertainty comparable to the $\sim 15\%$ uncertainty in the overall normalization due to the $SU(3)$-breaking effects in the quantity κ_u.

Neutral B-meson radiative decays are less-prone to the LD-effects, as argued above, and hence one expects that to a good approximation the ratio of the decay rates for neutral B meson obtained in the approximation of SD-dominance remains valid [7]:

$$\frac{\Gamma(B_d \to \rho\gamma, \omega\gamma)}{\Gamma(B \to K^*\gamma)} = \kappa_d \lambda^2[(1-\rho)^2 + \eta^2] \ . \tag{30}$$

Here κ_d represents the SU(3)-breaking effects in the form factor ratio squared. It is a realistic hope that this relation is theoretically (almost) on the same footing in the standard model as the one for the ratio of the B^0-$\overline{B^0}$ mixing-induced mass differences, which satisfies the relation [31]:

$$\frac{\Delta M_s}{\Delta M_d} = \kappa_{sd} \left|\frac{V_{ts}}{V_{td}}\right|^2 = \kappa_{sd} \frac{1}{\lambda^2[(1-\rho)^2 + \eta^2]} \ . \tag{31}$$

The hadronic uncertainty in this ratio is in the SU(3)-breaking factor $\kappa_{sd} \equiv (f_{B_s}^2 \hat{B}_{B_s} / f_{B_d}^2 \hat{B}_{B_d})$, which involves the pseudoscalar coupling constants and the so-called bag constants. This quantity is estimated as $\kappa_{sd} = 1.35 \pm 0.25$ in the QCD sum rules and lattice QCD approaches. (For details and references, see [31]). The present upper limit for the mass-difference ratio $\Delta M_s / \Delta M_d > 12.3$ at 95 % C.L. from the ALEPH data [37] provides better constraint on the CKM parameters, yielding $|V_{td}|/|V_{ts}| < 0.35$ than the corresponding constraints from the rare radiative decays $B \to (\rho, \omega) + \gamma$, which give an upper limit of 0.75 for the same CKM-ratio. We expect experimental sensitivity to increase in both measurements, reaching the level predicted for this ratio in the standard model, $|V_{td}|/|V_{ts}| = 0.24 \pm 0.05$ [31], in the next several years in the ongoing experiments at CLEO, LEP and Tevatron, and the forthcoming ones at the B factories and HERA-B.

Finally, combining the estimates for the LD- and SD-form factors in [19] and [7], respectively, and restricting the Wolfenstein parameters in the range $-0.4 \leq \rho \leq 0.4$ and $0.2 \leq \eta \leq 0.4$, as suggested by the CKM-fits [31], we give the following ranges for the absolute branching ratios:

$$\begin{aligned} \mathcal{B}(B_u \to \rho\gamma) &= (1.9 \pm 1.6) \times 10^{-6} \ , \\ \mathcal{B}(B_d \to \rho\gamma) &\simeq \mathcal{B}(B_d \to \omega\gamma) = (0.85 \pm 0.65) \times 10^{-6} \ , \end{aligned} \tag{32}$$

where we have used the experimental value for the branching ratio $\mathcal{B}(B \to K^* + \gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ [1], adding the errors in quadrature. The large error reflects the poor knowledge of the CKM matrix elements and hence experimental determination of these branching ratios will put rather stringent constraints on the

Summarizing the effect of the LD-contributions in radiative B decays, we note that they are dominantly given by the annihilation diagrams. QCD sum-rule-based estimates are very encouraging in that they lead to the conclusion that such contributions are modest in exclusive radiative B decays, in particular in the neutral B-decays $B^0 \to (\rho^0, \omega) + \gamma$. This estimate should be checked in other theoretically sound frameworks. Of course, forthcoming data on specific B-meson decays will be able to check this directly. Presently available data suggest that the contribution of annihilation diagrams in B decays is not significant, as seen through the near equality of the lifetimes for the $B^\pm$, B_d^0 and B_s^0 mesons and the near equality of the observed $B^\pm$ and B^0 radiative decay rates. We have argued that this is very probably also the case for the CKM-suppressed radiative decays, with $B^\pm \to \rho^\pm \gamma$ modified by $O(20)\%$ from its SD-rate.

Acknowledgements: I would like to thank Vladimir Braun, Christoph Greub, David London, and Hubert Simma for numerous helpful discussions and valuable input. Informative discussions with Arkady Vainshtein are also gratefully acknowledged.

References

[1] R. Ammar et al. (CLEO Collaboration), Phys. Rev. Lett. **71**, 674 (1993).

[2] M.S. Alam et al. (CLEO Collaboration), Phys. Rev. Lett. **74**, 2885 (1995).

[3] A. Ali and C. Greub, Z. Phys. **C49**, 431 (1991); Phys. Lett. **B259**, 182 (1991); Z. Phys., **C60**, 433 (1993).

[4] M. Ciuchini et al., Phys. Lett. **B316**, 127 (1993); Nucl. Phys. **B415**, 403 (1994); G. Cella et al., Phys. Lett. **B325**, 227 (1994).

[5] A.J. Buras et al., Nucl. Phys. **B424**, 374 (1994).

[6] A. Ali and C. Greub, Preprint DESY 95-117, SLAC-PUB-95-6940 (hep-ph/9506374).

[7] A. Ali, V. M. Braun and H. Simma, Z. Phys. **C63**, 437 (1994).

[8] P. Ball, TU-München Report TUM-T31-43/93 (1993);
P. Colangelo et al., Phys. Lett. **B317**, 183 (1993). S. Narison, Phys. Lett. **B327**, 354 (1994).

[9] M. Neubert, Phys. Rev. **D49**, 4623 (1994).

[10] I. Bigi et al., Phys. Rev. Lett. **71**, 496 (1993); Int. Journ. Mod. Phys. **A9**, 2467 (1994).

[11] G. Korchemsky and G. Sterman, Phys. Lett. **B340**, 96 (1994).

[12] A. Kapustin, Z. Ligeti and H.D. Politzer, Preprint CALT-68-2009 (hep-ph/9507248).

[13] A. Ali and E. Pietarinen, Nucl. Phys. **B154**, 519 (1979); G. Altarelli et al., Nucl. Phys. **B208,** 365 (1982).

[14] R.D. Dikeman, M. Shifman, and R.G. Uraltsev, Preprint TPI-MINN-95/9-T, UMN-TH-1339-95, UND-HEP-95-BIG05 (hep-ph/9505397).

[15] J. Chai, H. Georgi, and B. Grinstein, Phys. Lett. **B247**, 399 (1990);
I. Bigi, N. Uraltsev, and A. Vainshtein, Phys. Lett. **B293**, 430 (1992); **B297**, 477 (1993) (E);
A. Falk, M. Luke, and M. Savage, Phys. Rev. **D49**, 3367 (1994).

[16] E. Bagan, P. Ball, V.M. Braun, and H.G. Dosch, Phys. Lett. **B278**, 457 (1992);
M. Neubert, Phys. Rep. **245**, 259 (1994).

[17] P. Ball and V. Braun, Phys. Rev. **D49**, 2472 (1994); V.M. Braun (private communication).

[18] A. Ali and C. Greub , Phys. Lett. **B287**, 191 (1992).

[19] A. Ali and V.M. Braun, Preprint DESY-95-106 (hep-ph/9506248).

[20] A. Khodzhamirian, G. Stoll, and D. Wyler, Preprint ZU-TH-8-95 (hep-ph/9506242).

[21] T. Inami and C.S. Lim, Prog. Theor. Phys. **65**, 297 (1981).

[22] I. Bigi et al., Phys. Rev. **D50**, 2234 (1994);
M. Beneke and V.M. Braun, Nucl. Phys. **B426**, 301 (1994).

[23] P. Ball, M. Beneke and V.M. Braun, Preprint CERN–TH/95–65 [hep-ph/9503492].

[24] M. Shifman, N.G. Uraltsev, and A. Vainshtein, Phys. Rev. **D51**, 2217 (1995); M.B. Voloshin, Phys. Rev. **D51**, 4934 (1994).

[25] L. Gibbons (CLEO Collaboration), these proceedings.

[26] L. Montanet et al. (Particle Data Group), Phys. Rev. **D50**, 1173 (1994).

[27] V. Sudakov, Sov. Phys. JETP **3**, 65 (1956) ;
G. Altarelli, Phys. Rep. **81**,1 (1982).

[28] M. Jezabek and J.H. Kühn, Nucl. Phys. **B320**,20 (1989).

[29] G. Ricciardi, Phys. Lett. **B355**, 313 (1995).

[30] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[31] A. Ali and D. London, Preprint DESY 95-148, UdeM-GPP-TH-95-32 (hep-ph/9508272); to be published in Proc. of 6th. Int. Symp. on Heavy Flavour Physics, Pisa, Itlay, June 6 - 10, 1995.

[32] M. Athanas (CLEO Collaboration), CLEO CONF 94-2 (1994).

[33] J. M. Soares, Phys. Rev. **D49**, 283 (1994); S. Narison in [8].

[34] D. Atwood, B. Blok, and A. Soni, SLAC-PUB-95-6635 (hep-ph/9408373); N.G. Deshpande, X.-G. He, and J. Trampetic, Preprint OITS-564(1994); H.-Y. Cheng, Preprint IP-ASTP-23-94 (hep-ph/9411330); J.M. Soares, Preprint TRI-PP-95-6 (1995) (hep-ph/9503285); J. Milana, Preprint UMD-PP-95-110 (hep-ph/9503376); G. Eilam, A. Ioannissian, and R.R. Mendel, Preprint TECHNION-PH-95-4 (hep-ph/9505222).

[35] T. Browder, K. HonScheid, and S. Playfer, in: *B Decays*, 2nd. Edition, ed. S. Stone (World Scientific, Singapore, 1994).

[36] E. Golowich and S. Pakvasa, Phys. Rev. **D51**, 1215 (1995).

[37] D. Buskulic et al. (ALEPH Collaboration), CERN-PPE/95-084 (1995).